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Cracked rotor vibrations by multifractal analysis

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Abstract: Multifractal analysis has been used to diagnoze cracked and healthy rotors. Is has been shown that the complexity and regularity criteria of the dynamical systems defined by the multiple scaling of the time series can indicate the damages of the rotating shaft.

Keywords: nonlinear vibration, crack detection, health monitoring .

In many technical devices and machines possessing rotor, their actual dynamic condition determines their proper and safe operation. Most rotation machines operate over the extended periods of time in various temperature regions, and often they are subjected to large loads. As a consequence, of working conditions, their components are exposed to potential structural damage such as the shaft surface crack [1,2].

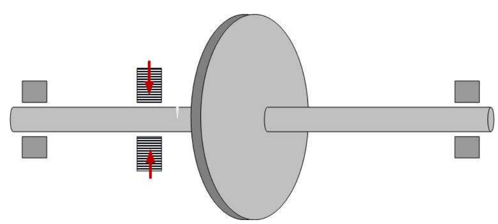

Figure 1. Schematic plot of the cracked rotor. In the experiment the shaft diameter was 15.875 mm and the shaft length was 0.659 m. The diameter of the active magnetic bearing rotors and radial actuator was 47.625 mm. The disk has a diameter of 127 mm and a thickness of 12.7 mm. The crack and magnetic actuator are depicted on the right hand side of the disk.

In the present paper we study the experimentally determined response of the test rotor system focusing on the effect of crack in a shaft. The test rotor consists of the shaft supported two ball bearings and a single disk located midspan on a flexible shaft (Fig. 1). An active magnetic actuator

placed near the disk produced an external harmonic force. The crack had the width of 0.94 mm and the depth of 40% of the shaft diameter. The experimental time series of measured displacement, in the direction of 45 deg from vertical one, for cracked and healthy rotors in the presence and a

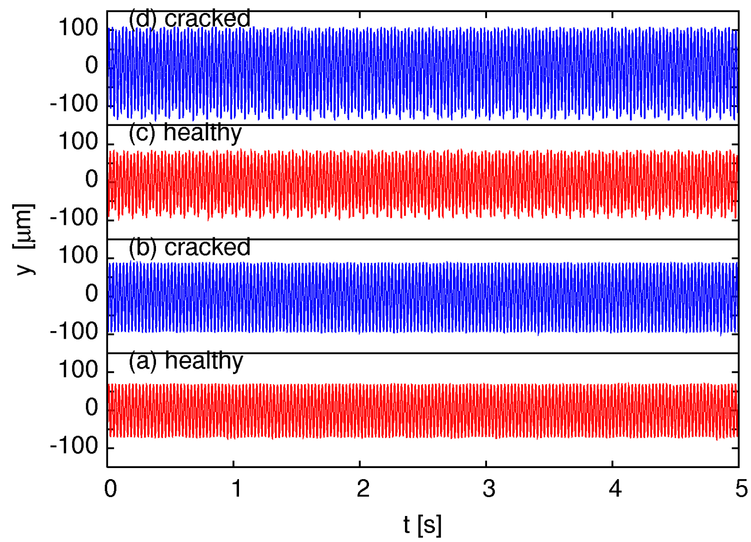

Figure 2. Experimental time series of the damaged and healthy machine for a spin velocity 2200 rpm (36.7 Hz): (a,b) without external excitation; (c,d) with magnetic actuator frequency 3780 rpm (63 Hz).

In Figs. 2a-b we plotted the results of unbalance for healthy (Fig. 2a) and cracked (Fig. 2b) rotors, respectively. One can see some increase of the amplitude. More sophisticated changes can be visible in Figs. 2c-d where healthy (Fig. 2c) and cracked (Fig. 2d) rotors have been subjected to the magnetic actuator applied harmonic force having amplitude of 200 N p-p (peak-to-peak) and fixed frequency [1].

In further studies we propose to use a multifractal analysis [3] which appeared to be a powerful tool to analyze the complexity of the nonlinear systems. This technique has been widely used in biological systems [3,4] but recently has been applied in engineering systems, e.g., to examine seismic sequences [5]. Following the multifractal procedure [3] we performed the Taylor expansion of the time series in the small vicinity time instant t_i we look for the exponent h_i (usually non-integer), which limits the error between the examined function and Taylor expansion and determines the local singularity in the time series:

$$y(t) - a_0 - a_1(t - t_i) - a_2(t - t_i)^2 - a_3(t - t_i)^3 - ... \leq a_h(t - t_i)^{h_i}.$$

Here a_i are locally determined Taylor expansion constants and a_h is a coefficient related to the exponent h_i. The multifractal analysis of rotor vibrations is based on constructing a singularity spectrum $f(h)$ of all h_i exponents providing a precise quantitative description of the system behaviour [3,4]. Formally, h defines the Hölder exponent while the probability of its distribution $f(h)$ coincides with the Hausdorff dimension of a dynamical system.

The results of our calculations are presented in Figs. 3a and b for the rotor response without external excitation and in the presence of magnetic actuators, respectively. The width of the spectrum $f(h)$, $\Delta h = h_{max} - h_{min}$ is defined as the complexity measure of the system response while the h_0 which corresponds to the maximum of $f(h)$ indicates the regularity of vibrations. The wider the range of possible fractal exponents, the "richer" the process, in a structure. Larger h_0 means more regular (or less stochastic) vibrations.

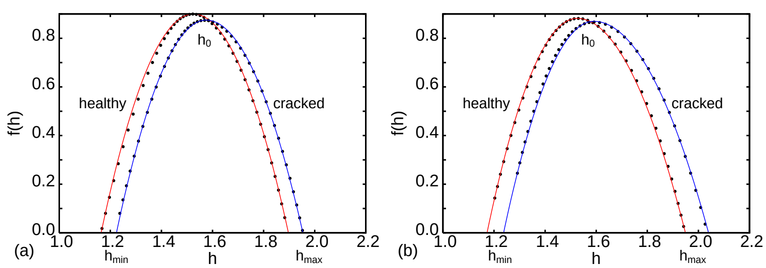

Figure 3. Spectra of the singularity exponents $f(h)$ for the system without any external excitation (a) and in the presence of magnetic actuators (b) (see the corresponding time series in Fig. 2). The width of corresponding spectra Δh and a peak position h_0 for healthy and cracked rotors: $\Delta h = 0.729$ & 0.732, $h_0 = 1.525$ & 1.575 (a); $\Delta h = 0.775$ & 0.803, $h_0 = 1.525$ & 1.590 (b).

In both plots we observe the shift of the spectrum to the right side (see the changes of h_0 position). This is related to more periodic behaviour of the cracked system. This effect is a manifestation of some extra coupling between vibration modes created by a defect in the rotor systems. Note also that in the rotor system without external excitation Fig. 3a. the changes in the multifractal results in terms of Δh are negligible (increase by 0.4 %) for the healthy and cracked rotors. Interestingly this difference of Δh, in case of the excited rotor, is noticeable (about one order higher increase - 3.6 %).

The above findings let us to conclude that by monitoring the changes in the multifractal spectra of a rotor we could identify crack (and other faults)

in a rotor at an early stage in their development. The presented results show that the use of an multifractal analysis can identify the complex response of the system. The role of the magnetic actuator is crucial to increase visibility of the crack–induced effect.

In summary, we conclude that the presented approach has some advantages, enabling to quantify the effect of crack using the measure of complexity. The alternative way is to analyze the corresponding Fourier spectrum [1]. However to produce a robust condition monitoring technique more tests are necessary. Especially one could perform different excitation type and procedure by using magnetic actuators. In our calculations we used the software provided by physionet [3].

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